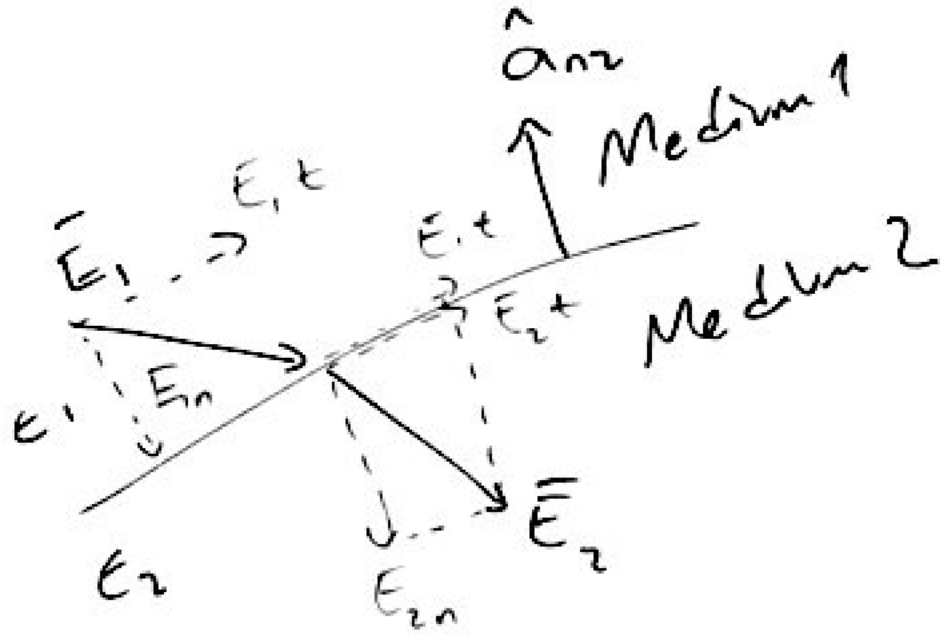
Boundary Conditions for Electrostatics:



1.) $\boxed{E_{1t} = E_{2t}}$ Tangential components of $\bar{E}$ are continuous across the interface.

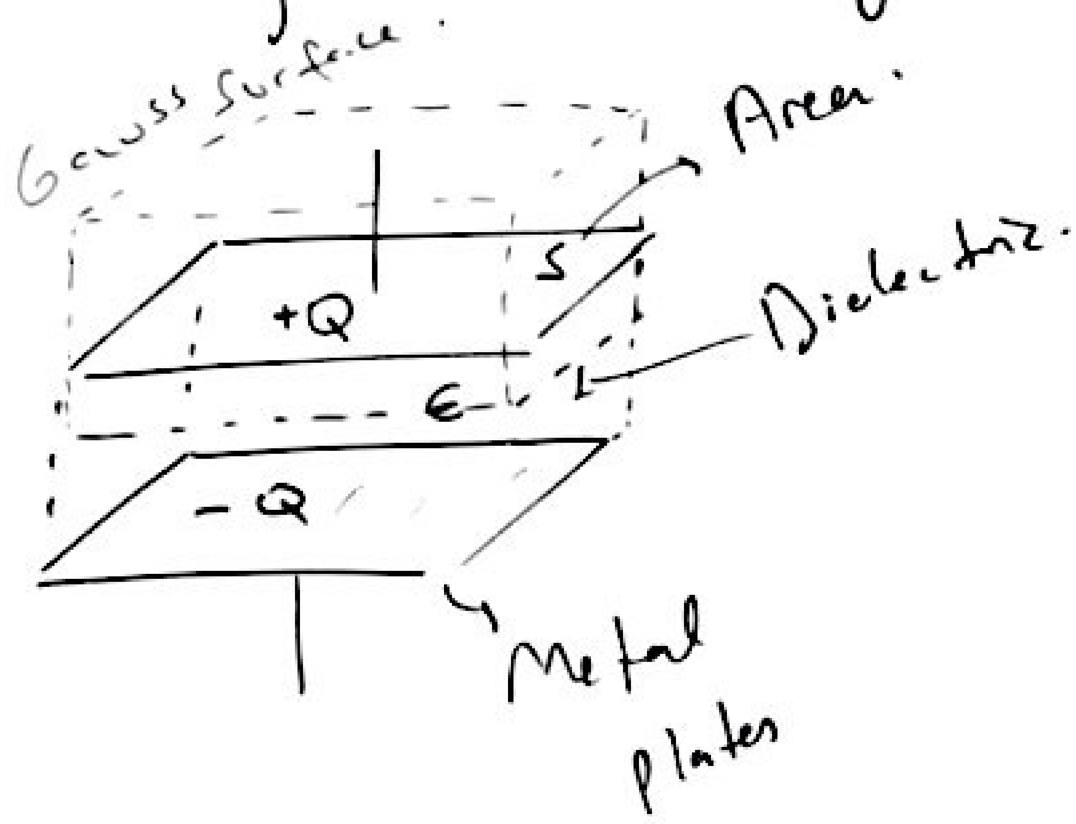
$$\boxed{\hat{a}_{n2} \cdot (\bar{D}_1 - \bar{D}_2) = \rho_s} \left(\frac{C}{m^2}\right)$$

or

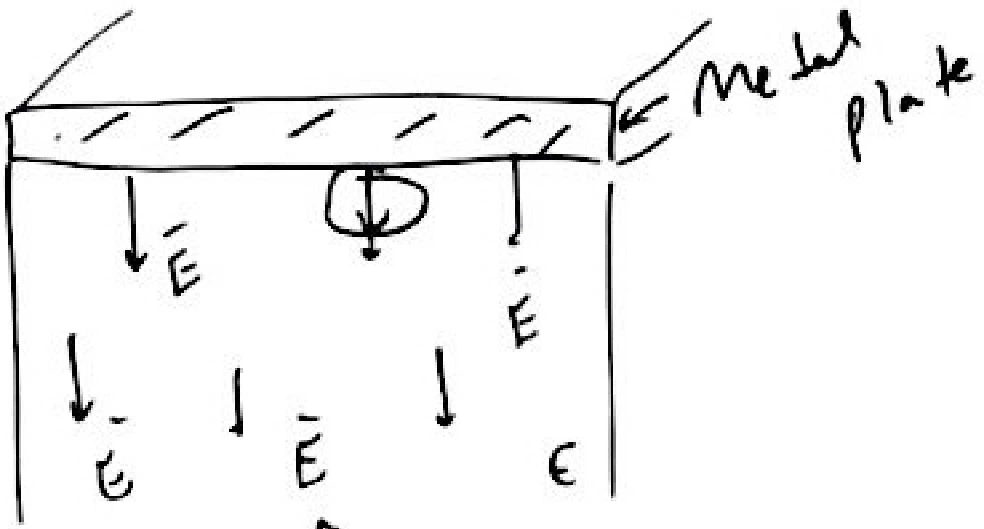
2.) $\boxed{D_{1n} - D_{2n} = \rho_s}$ ← Free surface charge density

Ex:

Find the $\bar{E}$-field in a parallel plate capacitor using the boundary condition.



Ans:

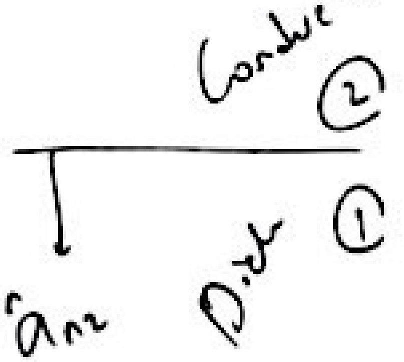


Assume that $\bar{E}$ is uniform and constant everywhere.
(we neglect the fringe field)

This assumption enables us to evaluate the $\bar{E}$-field everywhere by evaluating it at the boundary.

$$D_{1n} - D_{2n} = \rho_s \longrightarrow \rho_s = \frac{Q}{S}$$

$$\epsilon_1 E_1 - \epsilon_2 \underbrace{\cancel{E_2}}_{0} = \frac{Q}{S}$$



$$\epsilon_1 E_1 = \frac{Q}{S}$$

or

$$\boxed{E_1 = \frac{Q}{\epsilon_1 S}} \left(\frac{V}{m}\right)$$

2. Method (Gauss law):

$$\oint_S \bar{D} \cdot \bar{ds} = Q$$

$$\epsilon E . S = Q$$

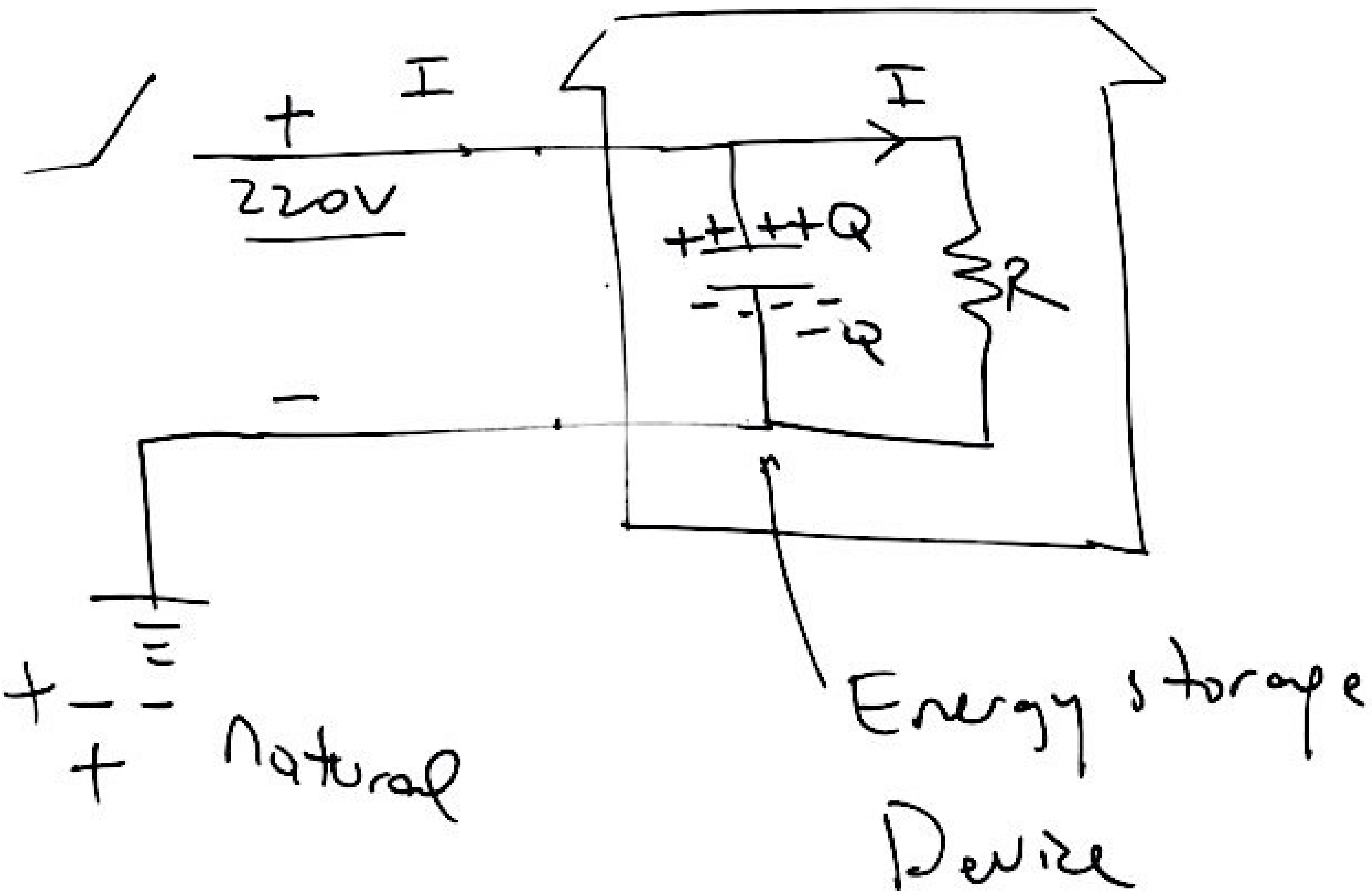
$$\boxed{E = \frac{Q}{\epsilon S}}$$

Capacitance and Capacitors:

charge

$$C = \frac{Q}{V} \quad \left(\frac{\text{Colomb}}{\text{volt}} = \text{Farad} \right)$$

voltage



Measure of this Energy Storage:

Capacitance = C.

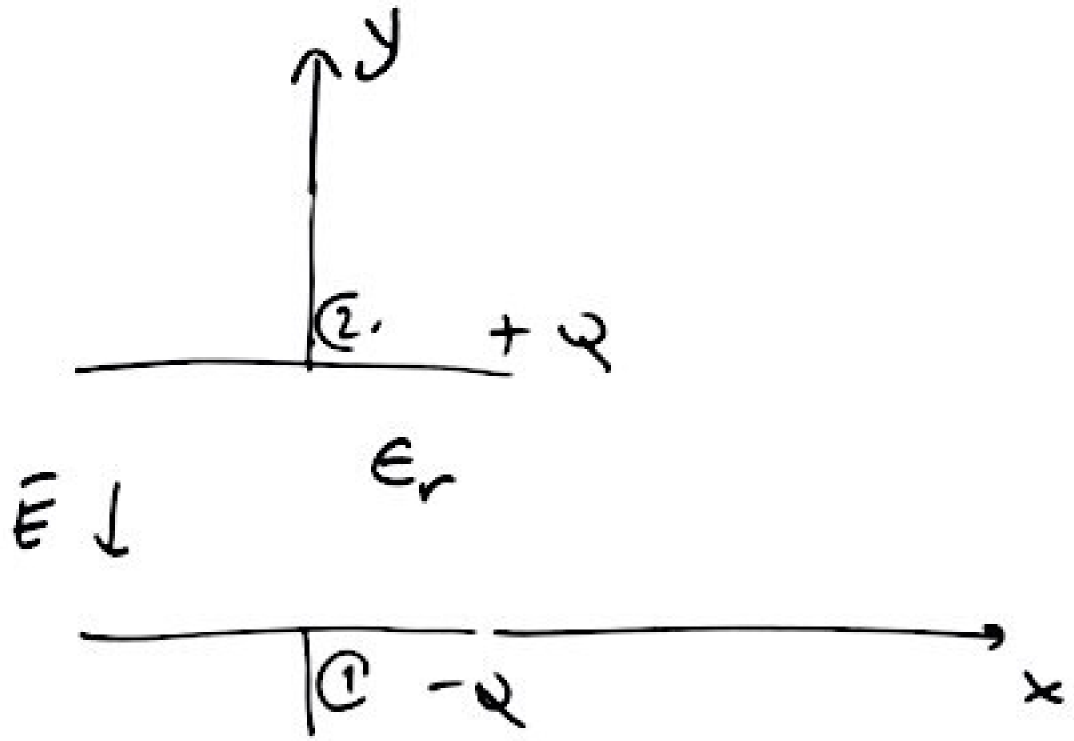
$$C = \frac{q}{V}$$ (Per volt, how much charge it can store is the capacitance.)

Capacitor is the name for an electrical device with capacitance.

<u>Ex:</u>

Determine the capacitance of a parallel plate capacitor.

Ans:



$$\bar{E} = -\hat{a}_y \frac{Q}{\epsilon S} \quad \text{(as before)}$$

$$V_{12} = -\int_{P_1}^{P_2} \bar{E} \cdot \underbrace{\bar{dl}}_{\hat{a}_y\, dy} = -\int_0^d \left(-\hat{a}_y \frac{Q}{\epsilon S}\right) \cdot (\hat{a}_y\, dy)$$

$$= \int_0^d \frac{Q}{\epsilon S}\, dy = \frac{Q\, d}{\epsilon S} \quad (V)$$

Then,

$$C = \frac{Q}{V} = \epsilon \frac{S}{d} \quad (F)$$

Steps to find Capacitance:

1-) Choose an appropriate coord. system for the given problem.

2-) Assume $+Q$ and $-Q$ charges on the conductors.

3-) Find $\bar{E}$

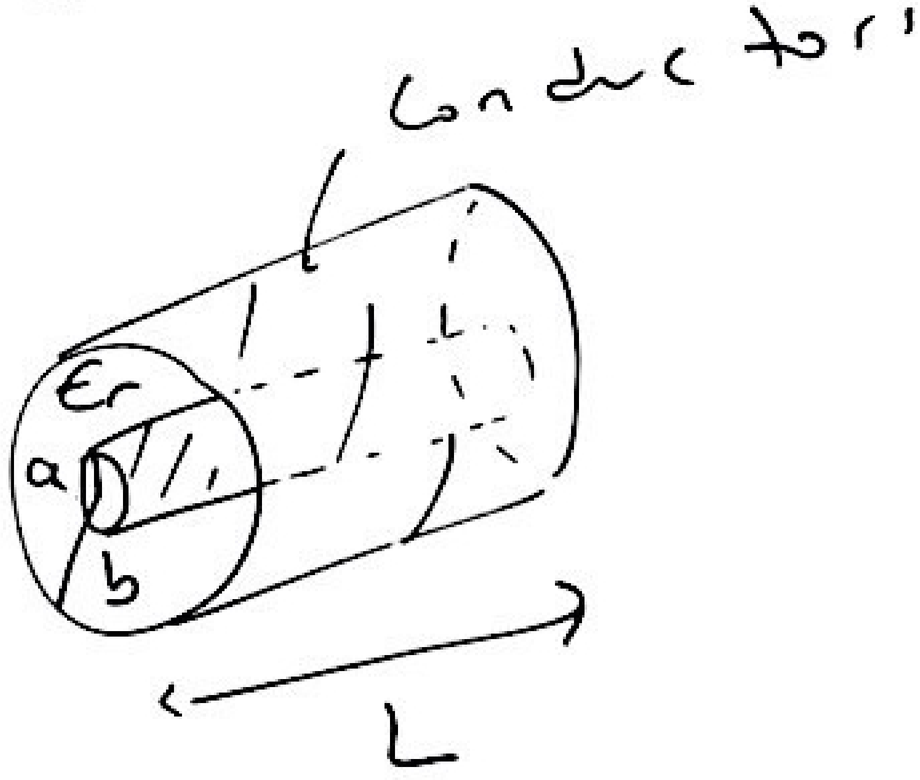
4-) Find V from $V = -\int_{\rho_1}^{\rho_2} \bar{E} \cdot \bar{dl}$.

5-) Find C from

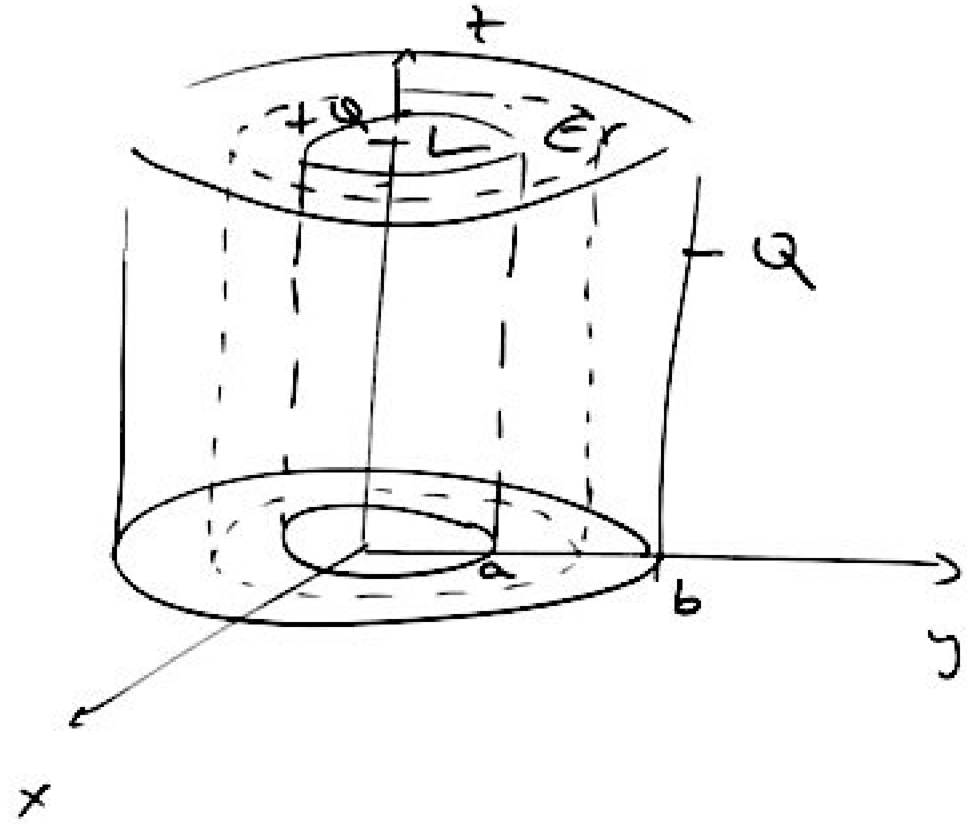
$$C = \frac{Q}{V}.$$

Ex:

Find the capacitance of a coaxial cable of length L, filled with dielectric of ϵ_r. given as



Ans:



The Electric field inside the dielectric is:

$$\bar{E}_r = \hat{a}_r E_r$$

$$E_r = \frac{Q}{2\pi \epsilon L r} \left(\frac{V}{m}\right)$$

$$V = -\int_{r=b}^{r=a} \bar{E} \cdot \bar{dl} = -\int_{b}^{a} \left(\hat{a}_r \frac{Q}{2\pi \epsilon L r}\right) \cdot \left(\hat{a}_r dr\right)$$

or

$$V = \frac{Q}{2\pi\epsilon L} \int_b^a \frac{1}{r}\, dr$$

or

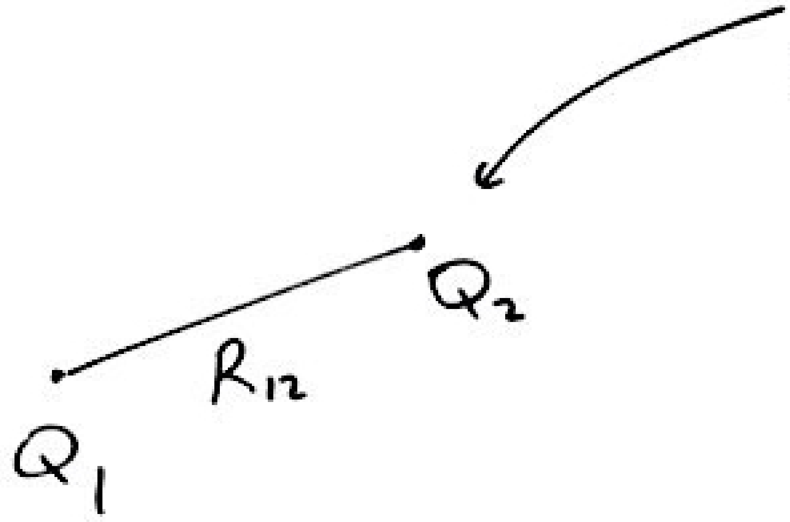
$$V = \frac{Q}{2\pi\epsilon L} \ln\left(\frac{b}{a}\right)$$

Then,

$$C = \frac{Q}{V} = \frac{2\pi\epsilon L}{\ln\left(\frac{b}{a}\right)} \quad (F)$$

Electrostatic Energy:

Bring Q_2 from infinity



$$\text{Work} = W_2 = Q_2 \overbrace{V_2}^{\frac{W}{Q}}$$

By definition, the voltage at a single point such as V_2 is defined as

$$V_2 = -\int_{\infty}^{R} \bar{E} \cdot \bar{dl} = -\int_{\infty}^{R} \left(\hat{a}_R \frac{Q}{4\pi\epsilon_0 R^2}\right) \cdot (\hat{a}_R\, dR)$$

$$= \frac{Q}{4\pi\epsilon_0 R} \; (V)$$

$$V_{12} = V_2 - V_1 = -\int_{P_1}^{P_2} \bar{E} \cdot \bar{dl}$$

$$W_2 = Q_2 \frac{Q_1}{4\pi\epsilon_0 R_{12}} = Q_1 \frac{Q_2}{4\pi\epsilon_0 R_{12}} = Q_1 V_1$$

or

$$W_2 = \frac{1}{2}(Q_1V_1 + Q_2V_2)$$

If we bring another charge Q_3 from infinity.

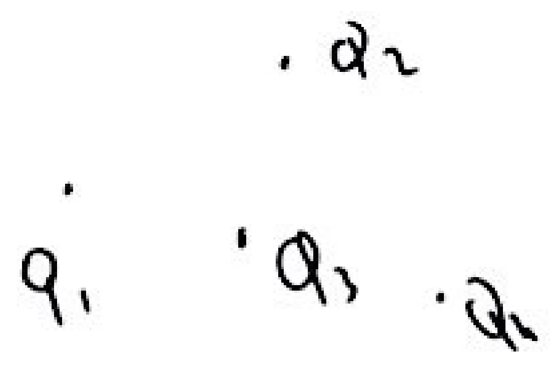
$\vdots$

$$W_3 = \frac{1}{2}(Q_1V_1 + Q_2V_2 + Q_3V_3)$$

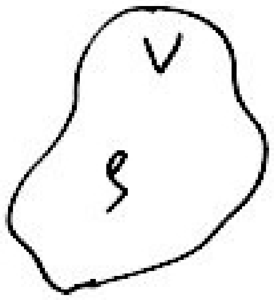
$\vdots$

$$W_e = \frac{1}{2}\sum_{k=1}^{N} Q_k V_k \quad \text{for } N \text{ charges}$$

This is the potential energy stored in the system.



If the charge distribution is continuous:



$$W_e = \frac{1}{2}\int_{v'} \rho V \, dv \quad (J)$$

In this equation

$$W_e = \frac{1}{2}\int_{v'} (\bar{\nabla}\cdot\bar{D}) V \, dv$$

Using the identity

$$\bar{\nabla}\cdot(V\bar{D}) = V\bar{\nabla}\cdot\bar{D} + \bar{D}\cdot\bar{\nabla}V$$

$$W_e = \frac{1}{2}\int_{v'} \bar{\nabla}\cdot(V\bar{D})\, dv - \frac{1}{2}\int_{v'} \bar{D}\cdot\bar{\nabla}V \, dv$$

$$= \underbrace{\cancel{\frac{1}{2}\oint_{s'} V\bar{D}\cdot\hat{a}_n \, ds}}_{0} + \frac{1}{2}\int_{v'} \bar{D}\cdot\bar{E} \, dv$$

Because:

As $R \to \infty$

$$V \propto \frac{1}{R}$$

$$D \propto \frac{1}{R^2}$$

Thus,

$$W_e = \frac{1}{2} \int_{v'} \overset{\epsilon \bar{E}}{\bar{D}} \cdot \bar{E} \, dv$$

or

$$W_e = \frac{1}{2} \int_{v'} \epsilon E^2 \, dv$$

or

$$W_e = \frac{1}{2} \int_{v'} \frac{D^2}{\epsilon} \, dv$$

Here

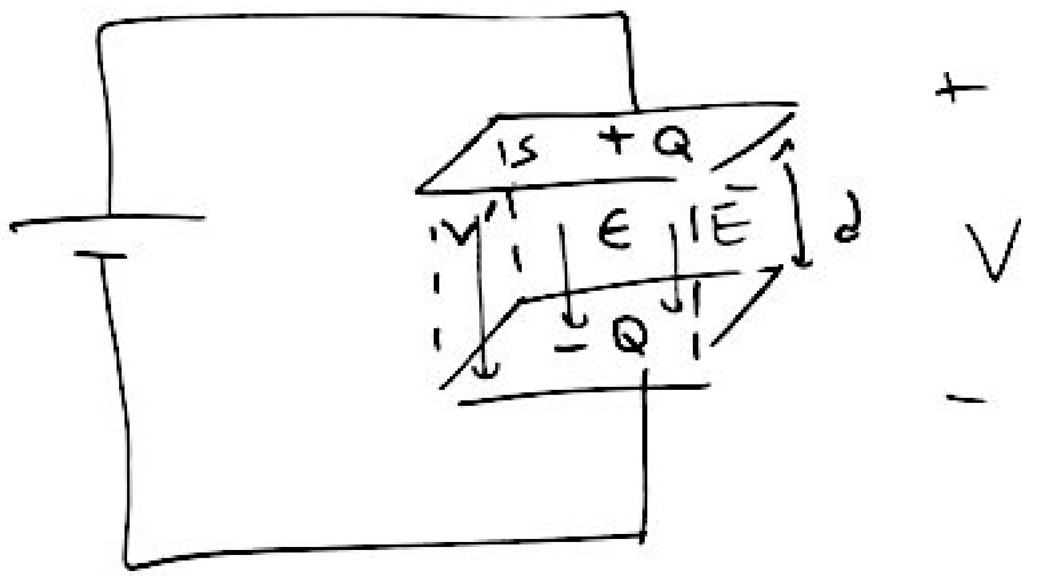
$$w_e = \text{energy density} = \frac{1}{2} \epsilon E^2 \quad \left(\frac{J}{m^3}\right)$$

Ex:

Parallel plate capacitor of area S and seperation d is charged to a voltage V.

The permittivity of the dielectric is ϵ.

Find the stored electrostatic energy



Ans:

$$E = \frac{V}{d}$$

$$W_e = \frac{1}{2}\int_{v'} \epsilon \left(\frac{V}{d}\right)^2 dv' = \frac{1}{2}\epsilon\left(\frac{V}{d}\right)^2 (Sd)$$

$$= \frac{1}{2}\underbrace{\left(\epsilon\frac{S}{d}\right)}_{C} V^2$$

$$\boxed{W_e = \frac{1}{2} C V^2 \quad (J)}$$

Since $Q = CV$

$$\boxed{W_e = \frac{1}{2} Q V \quad (J)}$$

or

$$\boxed{W_e = \frac{Q^2}{2C} \quad (J)}$$

Chp. 5. Steady Electric Currents:

$$I = \frac{Q}{t} \quad \left(\frac{C}{sec} = \text{Ampere}\right)$$

The current density $\bar{J}$ is defined as

$$\bar{J} = \rho \vec{v} \quad \left(\frac{A}{m^2}\right)$$

where

ρ = Charge density $\left(\frac{C}{m^3}\right)$

and

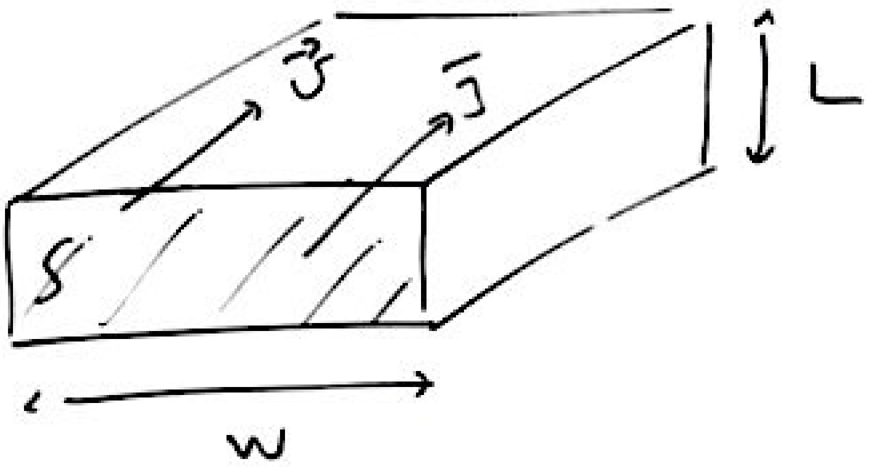
$\bar{v}$ = Charge velocity $\left(\frac{m}{sec}\right)$

The current density $\bar{J}$ is related to the current I as

$$I = \int_S \bar{J} \cdot \bar{ds}$$

Ex:

In the given figure calculate the current in terms of the current density $\bar{J}$.



Assuming J is steady and uniform

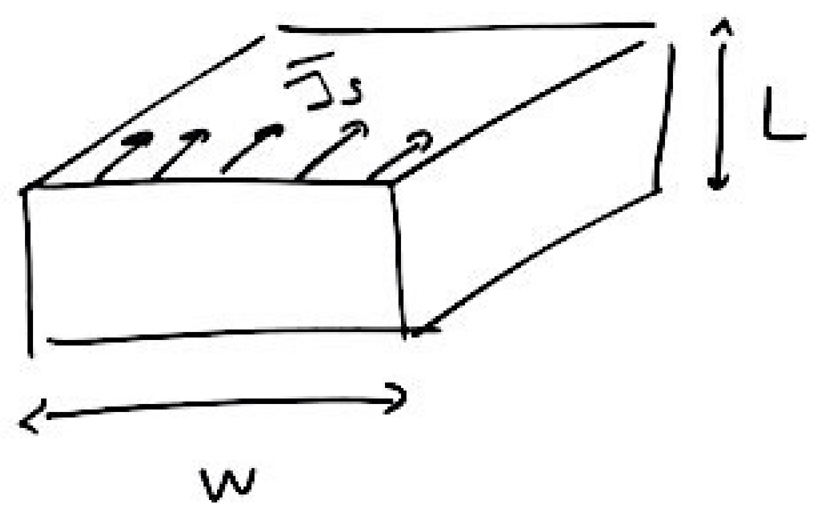
not changing with time

The same everywhere.

Ans:

$$I = \int_S \bar{J} \cdot \bar{ds} = W \cdot L \cdot J \ (A)$$

Surface Current density J_s $\left(\frac{A}{m}\right)$



Assume a Uniform $\bar{J}_s$ on the top surface.

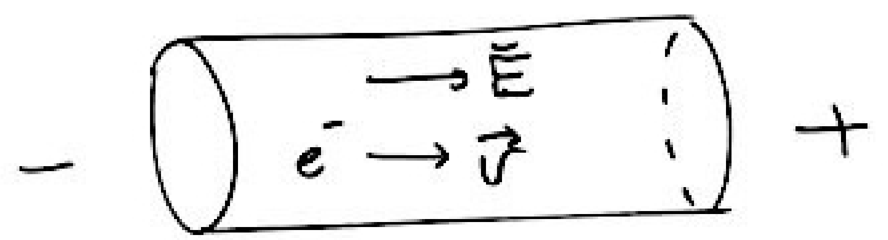
$$\boxed{I = \int_c \bar{J}_s \cdot d\bar{l}}$$

$$I = L \cdot J_s \ (A)$$

For most conducting materials,

$$\vec{v} = -\mu_e \bar{E}$$

where μ_e = Mobility of electrons. $(m^2/V.s)$



We have

$$\bar{J} = \rho \vec{v} = -\rho \mu_e \bar{E}$$

or

$$\boxed{\bar{J} = \sigma \bar{E} \quad \left(\frac{A}{m^2}\right)}$$

where

σ = Conductivity $\left(\frac{S}{m}\right)$

$\sigma_{Copper} = 5.8 \times 10^{7} \left(\frac{S}{m}\right)$ Conductor

$\sigma_{Germanium} = 2.2 \left(\frac{S}{m}\right)$

$\sigma_{Silicon} = 1.6 \times 10^{-3} \left(\frac{S}{m}\right)$

$\left.\right\}$ Semi-Conductor

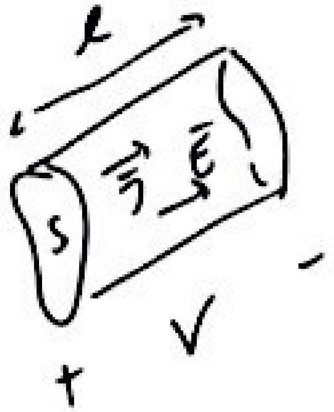
$\sigma_{Rubber} = 10^{-15} \left(\frac{S}{m}\right)$ $\left.\right\}$ Insulator

Define

$$\frac{1}{\sigma} = \rho = \text{Resistivity } (\Omega \cdot m)$$

Ex:

Find the Resistance of a conductor



Uniform field distributions are assumed.

Then,

$$V = E \cdot l$$

or

$$E = \frac{V}{l}$$

and

$$I = \int_S \bar{J} \cdot d\bar{S} = J S$$

or

$$J = \frac{I}{S}$$

Now, we have

$$\bar{J} = \sigma \bar{E}$$

or

$$J = \sigma E$$

or

$$\frac{I}{S} = \sigma \frac{V}{l}$$

or

$$V = \underbrace{\left(\frac{l}{\sigma S}\right)}_{R} I$$ (Ohm's law in volumetric form)

Then,

$$R = \frac{l}{\sigma S} \quad (\Omega)$$

Power Dissipation:

$$p = \lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = q\bar{E} \cdot \vec{u}$$

$\bar{E} = \frac{F}{q}$, $\vec{u} = \frac{\Delta x}{\Delta t}$

(power correspond to moving a charge q against an electric field E)

The power delivered to all charges in volume dv is

$$dP = \sum p = E \cdot \left(\sum q\vec{v}\right) dv$$

or $dp = \bar{E} \cdot \bar{J} \, dv$

Integrating both sides in volume

V

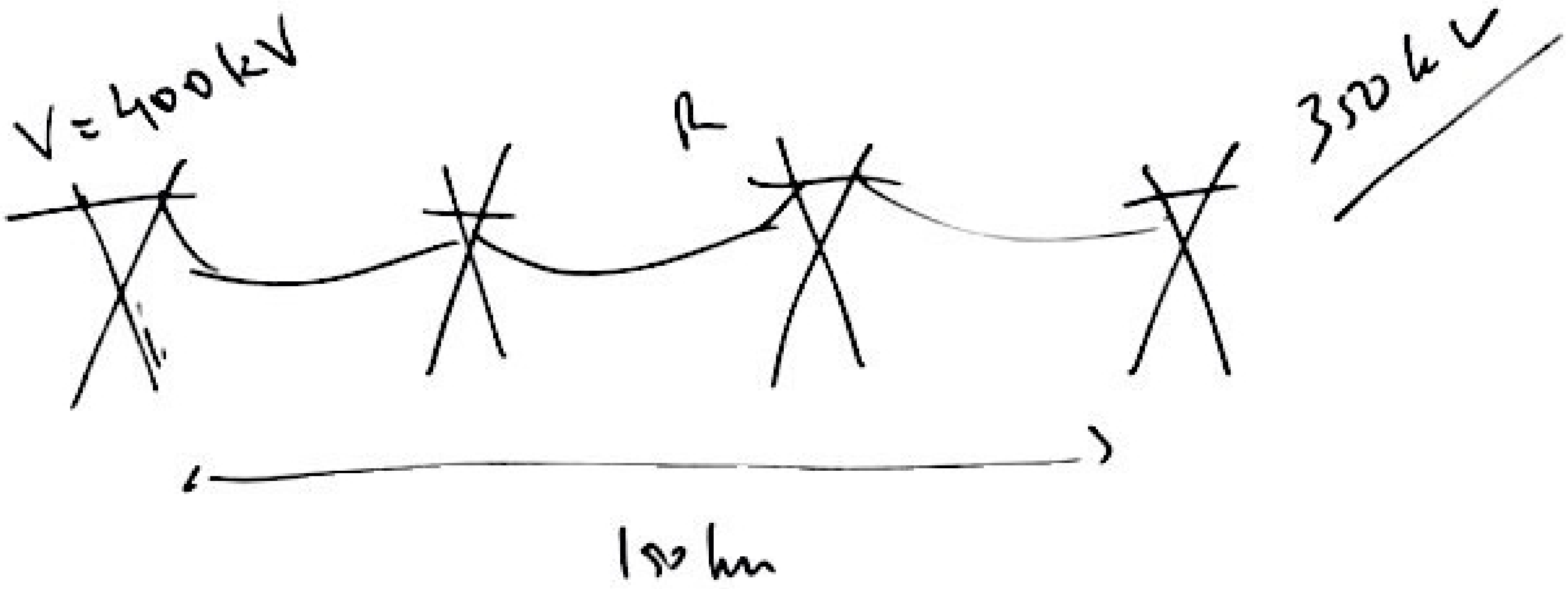
$$\boxed{P = \int_V \bar{E} \cdot \bar{J} \, dv \quad (W)}$$

$$= \int_L E \, d\bar{l} \int_S \bar{J} \cdot d\bar{s} = V \cdot I \quad (W)$$

(I R)

or

$$\boxed{P = I^2 R \quad (W)}$$



Equation of Continuity:

This equation gives the relation between the current density and charge.

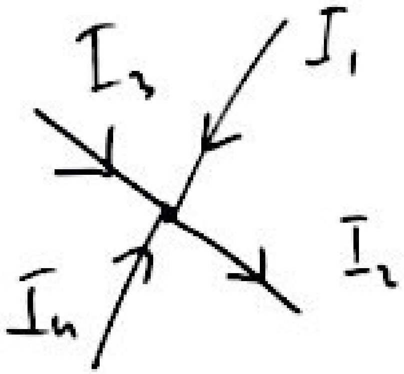
$$\boxed{\bar{\nabla} \cdot \bar{J} = -\frac{\partial \rho}{\partial t}}$$

For steady currents

$$\frac{\partial}{\partial t} = 0$$

Then,

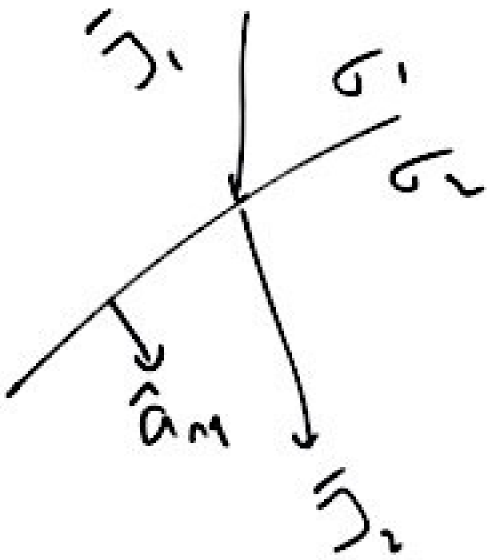
$$\bar{\nabla} \cdot \bar{J} = 0$$



Take the integral of both sides

$$\boxed{\oint_S \bar{J} \cdot \bar{ds} = 0} \quad \text{or} \quad \boxed{\sum_j I_j = 0 \quad (A)}$$ (Kirchoff's current law)

Boundary Conditions for Current Density:



$$\boxed{J_{1n} = J_{2n}}$$

$$\boxed{\frac{J_{1t}}{J_{2t}} = \frac{\sigma_1}{\sigma_2}}$$

Resistance Calculations:

Steps:

1-) Choose the coord. system.

2-) Assume a potential difference V_o between conductor terminals.

3-) Find $\bar{E}$ within the conductor.

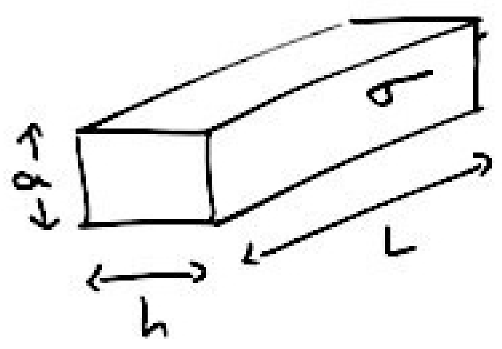
4-) Find the total current

$$I = \int_s \bar{J} \cdot d\bar{s} = \int_s \sigma \bar{E} \cdot d\bar{s}$$

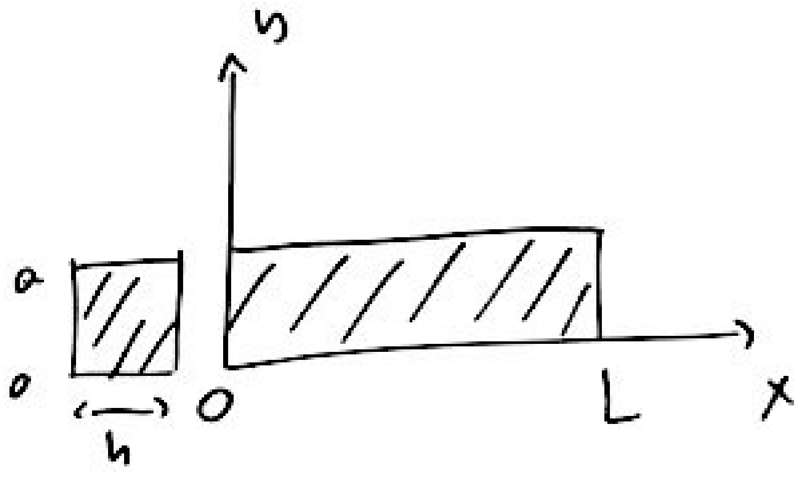
5-) Find the resistance from $R = \dfrac{V_o}{I}$.

Ex:

Find the resistance of the material



Ans:



$V = 0$ at $x = 0$

$V = V_0$ at $x = L$

$E = E(x)$ only.

In order to find E:

$$\bar{\nabla} \cdot \bar{D} = \rho$$

or

$$\bar{\nabla} \cdot \epsilon \bar{E} = \rho$$

or

$$\bar{\nabla} \cdot (\epsilon \bar{\nabla} V) = -\rho$$

$$\nabla \cdot (\bar{\nabla} V) = \frac{-\rho}{\epsilon}$$

or

$$\boxed{\nabla^2 V = -\frac{\rho}{\epsilon}}$$ (Poisson's equation)

∇^2 is called the Laplacian operator

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$ (in cartesian coord.)

Since we want to find the solution of the $\bar{E}$-field within the conducting material, we assume the free charge density $\rho = 0$ inside the material.

Thus, we need to solve

$$\nabla^2 V = 0 \quad \text{(Laplace equation)}$$

with boundary conditions

$$\boxed{\begin{array}{l} V = 0 \text{ at } x = 0 \\ \text{and} \\ V = V_0 \text{ at } x = L \end{array}}$$

$\nabla^2 V = 0$ gives

$$\boxed{\frac{\partial^2 V}{\partial x^2} = 0}$$

The general solution is

$$V(x) = C_1 x + C_2$$

where C_1 and C_2 are constants to be determined by the boundary conditions.

$$C_2 = 0 \quad , \quad C_1 = \frac{V_0}{L}$$

$$V(x) = \frac{V_0}{L} x$$

$$\bar{E} = -\bar{\nabla} V = -\hat{a}_x \frac{V_0}{L}$$

$$\bar{J} = \sigma \bar{E} = -\sigma \bar{\nabla} V = -\hat{a}_x \frac{\sigma V_0}{L}$$

$$I = \int_S \bar{J} \cdot d\bar{s} = \int_S \left(-\hat{a}_x \frac{\sigma V_0}{L}\right) \cdot \left(-\hat{a}_x \, dy \, dz\right)$$

$$= \frac{\sigma V_0}{L} \cdot (ah) \quad , \quad R = \frac{V_0}{I} = \frac{V_0}{\sigma \frac{V_0}{L} (ah)} = \frac{L}{\sigma \underbrace{(ah)}_{S}} \; (\Omega)$$

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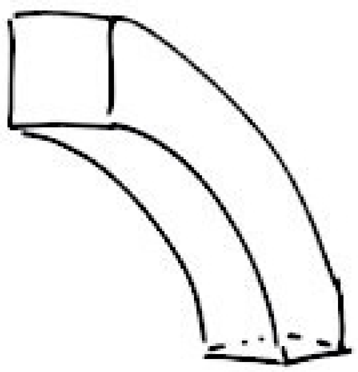
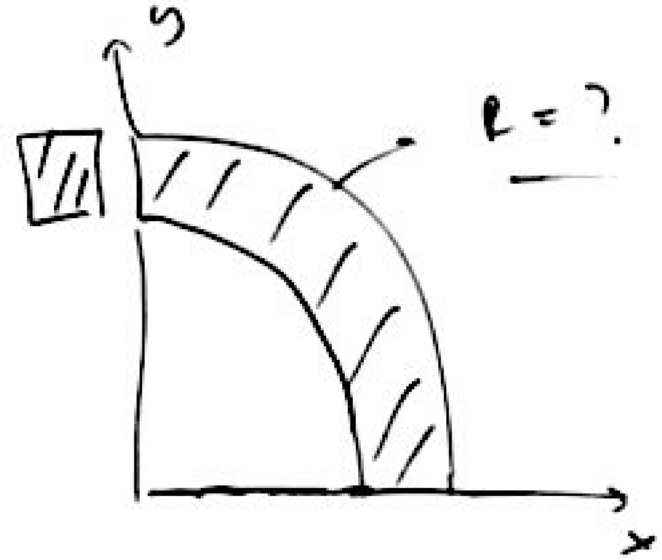
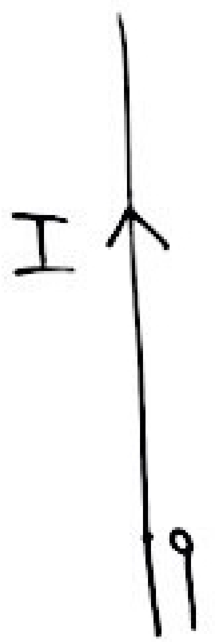




Cp. 6. Static Magnetic Fields:

$$\frac{\partial}{\partial t} = 0$$



q_t $\bar{F}_m$

$\bar{B}$ = Magnetic Field $\left(\frac{Wb}{m^2}\right)$
Density Vector

$\bar{E}$ $\bar{F}_e = \bar{E} \cdot q_t$

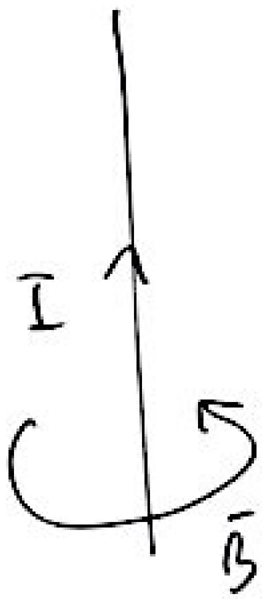
q_t

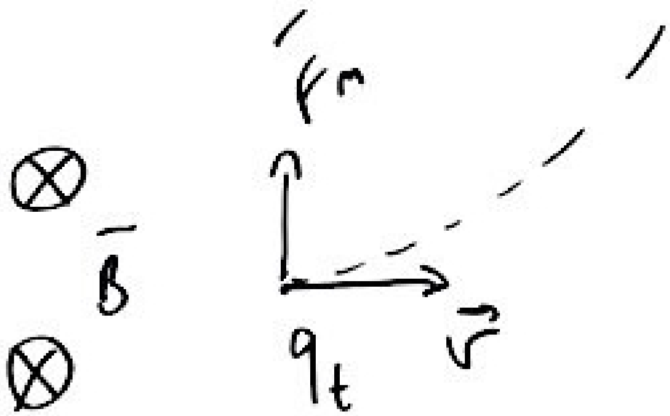
$$\vec{F}_m = q_t \vec{v} \times \vec{B}$$

velocity vector.

$$\boxed{\oint_S \bar{E} \cdot d\bar{s} = \frac{Q}{\epsilon}}$$

$$E = \int_{v'} f\Big|_{r=R} dv'$$





$$\bar{F}_{total} = \bar{F}_e + \bar{F}_m$$
$$= \bar{E} q + q \vec{v} \times \vec{B}$$
$$= q(\bar{E} + \vec{v} \times \vec{B}) \quad \text{(Lorentz equation)}$$

$$\boxed{1 \frac{Wb}{m^2} = 1\,\text{Tesla} = 10^4\,\text{Gauss}}$$

Postulates for Magnetostatics.

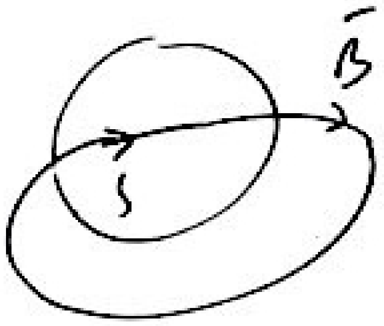
$$\bar{\nabla}\cdot\bar{B} = 0$$

$$\bar{\nabla}\times\bar{B} = \mu_0 \bar{J}$$

Integral forms are

$$\oint_S \bar{B}\cdot d\bar{s} = 0$$

Mag. field lines always close upon themselves.



Stokes' theorem

$$\int_S (\bar{\nabla}\times\bar{B})\cdot d\bar{s} = \mu_0 \int_S \bar{J}\cdot d\bar{s}$$

$$I$$

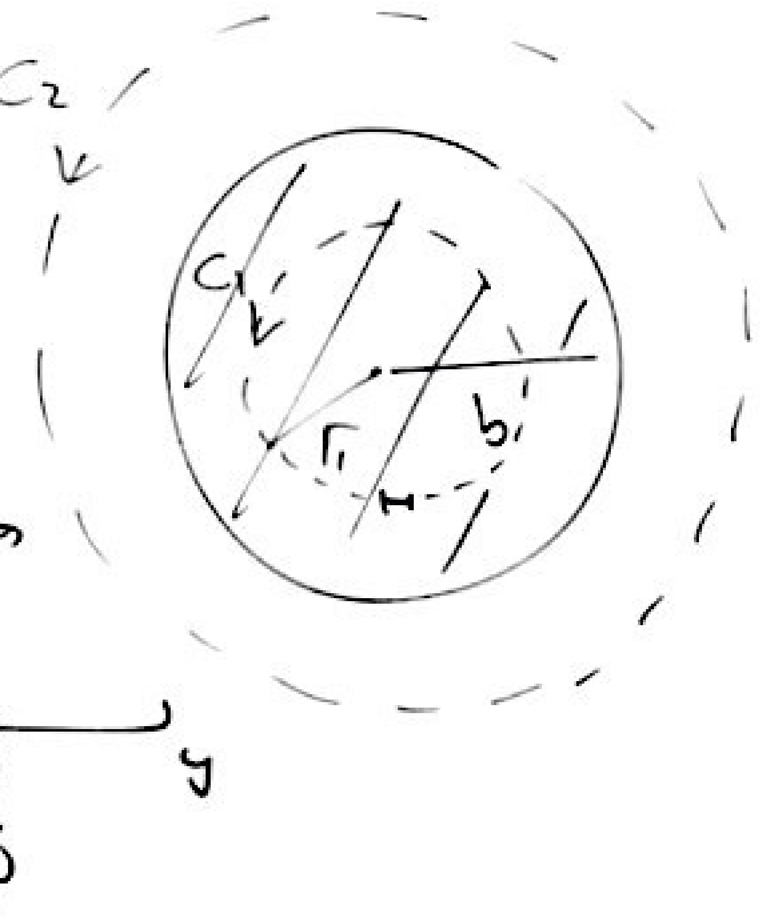
$$\boxed{\oint_C \bar{B}\cdot d\bar{l} = \mu_0 I}$$

(Ampere's law)

Ex: 6.1

An infinitely long, straight conductor with a circular cross section of radius b carries a steady current I. Determine the magnetic flux density both inside and outside the conductor.

Ans:



Inside the Conductor:

$$\bar{B}_1 = \hat{a}_\phi B_{\phi_1}$$

$$\oint_{C_1} \bar{B}\cdot d\bar{l} = \mu_0 I$$

The current passing through the contour C_1.

$$\int_0^{2\pi} (\hat{a}_\phi B_{\phi_1})\cdot(r_1 \, d\phi) = \mu_0 \underbrace{\left(\frac{r_1}{b}\right)^2 I}_{\text{The current inside the circl}}$$

$$\boxed{\bar{B}_1 = \hat{a}_\phi B_{\phi_1} = \hat{a}_\phi \frac{\mu_0 r_1 I}{2\pi b^2}} \quad r_1 \leq b$$

Outside the conductor:

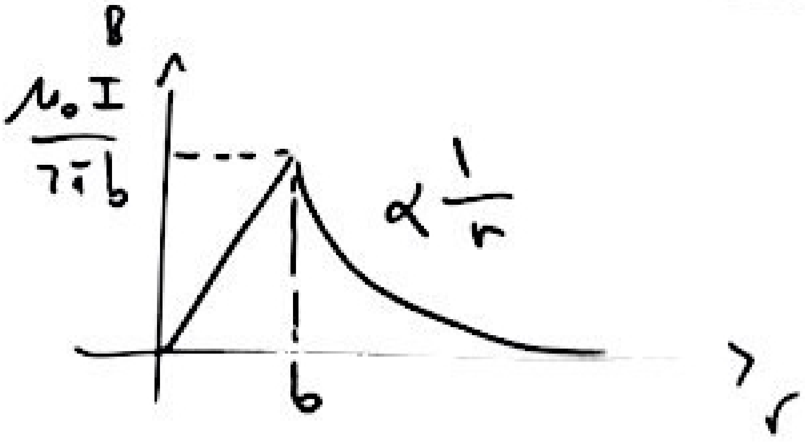
$$\bar{B}_2 = \hat{a}_\phi B_{\phi_2} \qquad \bar{dl} = \hat{a}_\phi r_2 \, d\phi$$

$$\oint_{C_2} \bar{B}_2 \cdot \bar{dl} = 2\pi r_2 B_{\phi 2}$$

$$2\pi r_2 B_{\phi_2} = \mu_0 I$$

Thus,

$$\boxed{B_{\phi 2} = \frac{\mu_0 I}{2\pi r_2} \left(\frac{wb}{m^2}\right)} \quad r_2 \geq b$$



Magnetic Field!

This is also a force field.

$\bar{F}_m = q\vec{v} \times \bar{B}$

↳ Magnetic flux density vector $\left(\frac{wb}{m^2}\right)$

Two equations for static magnetic field:

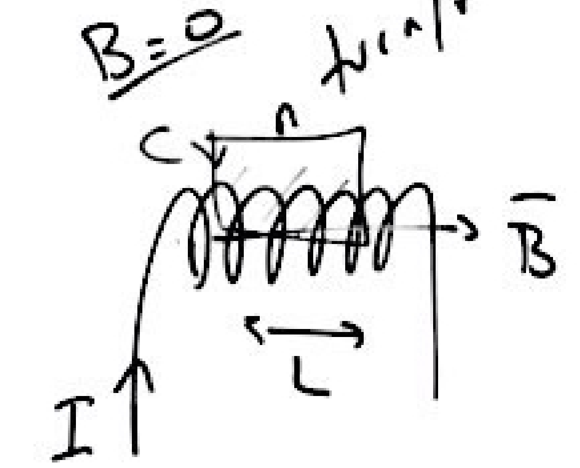
$\bar{\nabla} \cdot \bar{B} = 0 \longrightarrow \oint_S \bar{B} \cdot \bar{ds} = 0$

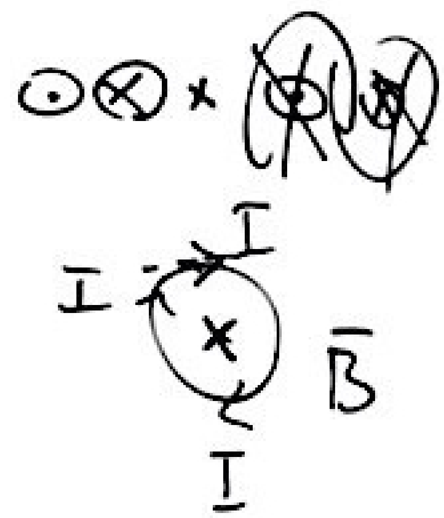
$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J} \rightarrow \oint_C \bar{B} \cdot \bar{dl} = \mu_0 I$ (Ampere's law)

Ex:

Determine the magnetic field density vector $\bar{B}$ inside an infinitely long solenoid with air core having n turns per unit length and carrying a current I.

Ans:





$$\oint_C \bar{B}\cdot d\bar{l} = \int_0^L (\hat{a}_x B_x)\cdot(\hat{a}_x dx) = \mu_0 n L I$$

$$B_x \int_0^L dx = \mu_0 n L I \Rightarrow \boxed{B_x = \mu_0 n I \ \frac{Wb}{m^2}}$$

If

$\mu_0 = 4\pi \times 10^{-7}$

$I = 10A$

$n = ?$

$B = 20.000$ Gauss

$\left(1 \frac{Wb}{m^2} = 10^4 \text{ Gauss}\right.$

$\Rightarrow 20.000 \text{ Gauss} = 2 \frac{Wb}{m^2} \Big)$

Thus

$$n = \frac{\cancel{2}}{\cancel{4}\pi \times 10^{\cancel{-7}\,-6} \times \cancel{10}} = \frac{1}{6} \times 10^6$$

$$\cong 0.17 \times 10^6$$

$$\cong 170 \times 10^3$$

$\cong 170000$ turns per unit length

$\sigma \rightarrow \boxed{\frac{1}{\sigma} = \delta}$

$\bar{J} = \sigma \bar{E}$

Power loss $= \int_v \bar{E} \cdot \bar{J} \cdot dv = \int_v \sigma |E|^2 dv = I^2 \cdot R$ (W)

170.000 turns per meter is very difficult.

Vector Magnetic Potential:

It's defined by

$$\bar{B} = \bar{\nabla} \times \bar{A}$$

where

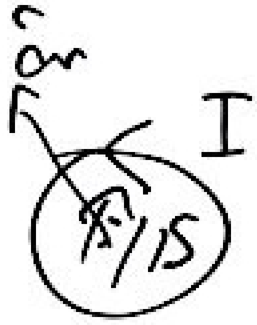
$\bar{A}$ = Vector magnetic potential

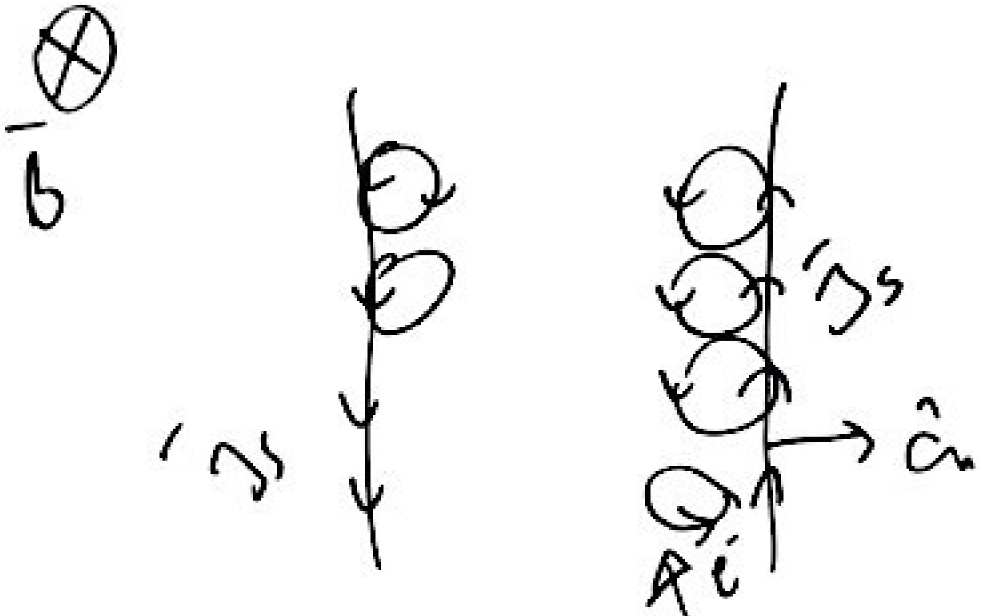
This concept is similar to electric potential (V).

In integral evaluations of the electric field, the voltage (V) is easier to find since it depends upon $\frac{1}{r}$ and the electric field $\propto \frac{1}{r^2}$.

In calculating the magnetic field, it's also easier to use the vector potential $\bar{A}$ than $\bar{B}$.

Magnetization





If magnetized

$\bar{m}$ = magnetic dipole moment.

$= \hat{a}_n I S$

$\bar{M} = \frac{\Sigma \bar{m}}{dv}$ (Magnetization vector)

$$\boxed{\begin{aligned} \bar{J}_m &= \bar{\nabla} \times \bar{M} \\ \bar{J}_s &= \bar{M} \times \hat{a}_n \end{aligned}}$$

$$\frac{1}{\mu_0} \bar{\nabla} \times \bar{B} = \bar{J} + \bar{J}_m = \bar{J} + \bar{\nabla} \times \bar{M}$$

$$\bar{\nabla} \times \Big(\underbrace{\frac{\bar{B}}{\mu_0} - \bar{M}}_{\bar{H}} \Big) = \bar{J}$$

$\bar{H}$ = Mag. field Intensity vector.

$$\bar{\nabla} \times \bar{H} = \bar{J}$$

$$\boxed{\oint_c \bar{H} \cdot \bar{dl} = I}$$ (Ampere's law general form)

$\bar{M} = \chi_m \bar{H}$

↳ magnetic susceptibility

$\bar{B} = \mu_0 (1 + \chi_m) \bar{H}$

$= \mu_0 \mu_r \bar{H} = \mu \bar{H}$

↳ permeability

$\boxed{\bar{B} = \mu \bar{H}}$ $\boxed{\bar{H} = \frac{\bar{B}}{\mu}}$

Diamagnetic Materials:

$\mu_r \lesssim 1$

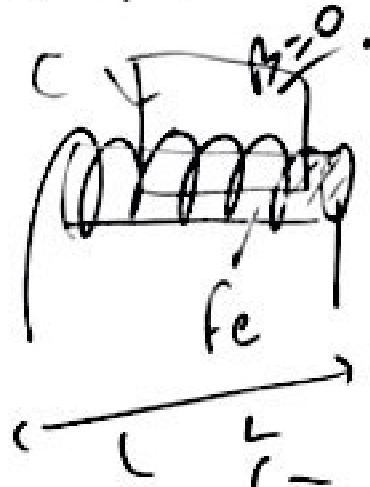
Paramagnetic : $\mu_r \gtrsim 1$

Ferromagnetic : $\mu_r >> 1$

↳ Ferrites:

Ex:

Fe (iron) $\mu_r = 7000$.



Solenoid.

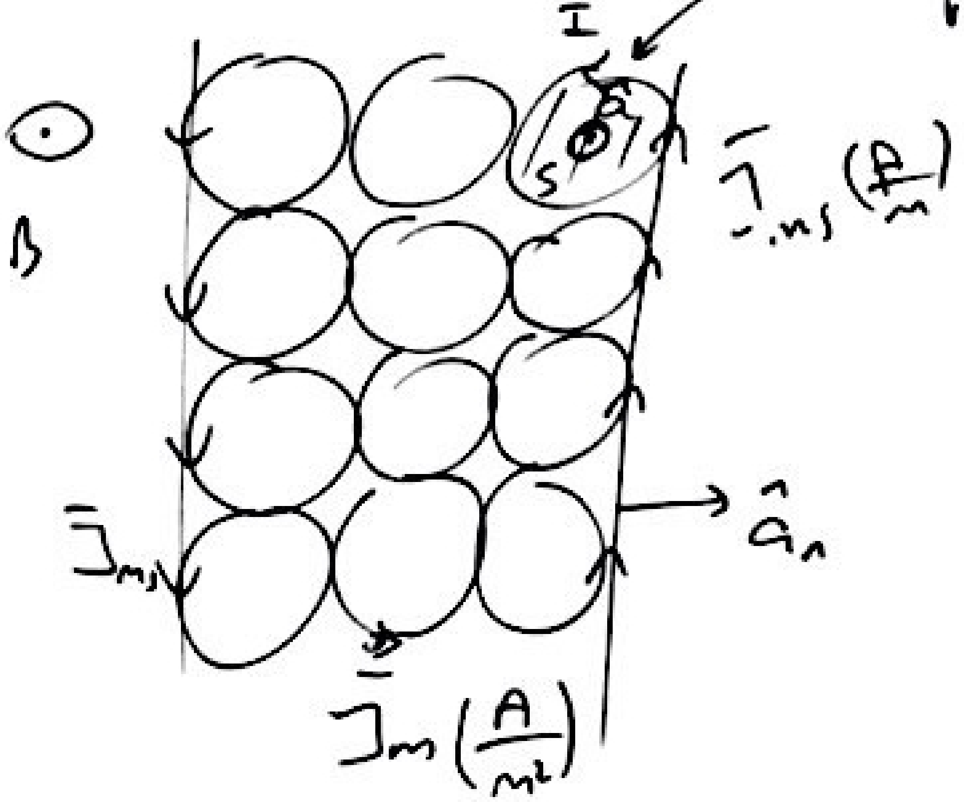
$$\oint \bar{H} \cdot d\bar{l} = nLI$$

$$= \frac{B \cdot L}{\mu_r \mu_0} = nLI = \mu_r \mu_0 n I \left(\frac{wb}{m^2}\right)$$

Magnetization:

When magnetized materials are put inside a magnetic field, magnetization occurs.

magnetic dipole moment $= \bar{m} = \hat{a}\, I \cdot s$



$\bar{M}$ = Magnetization vector

$= \dfrac{\sum \bar{m}}{\Delta v}$ (magnetic dipole moment volume density)

$$\bar{J}_m = \bar{\nabla} \times \bar{M}$$

$$\bar{J}_{ms} = \bar{M} \times \hat{a}_n$$

$$\frac{1}{\mu_0}\bar{\nabla}\times\bar{B} = \bar{J} + \bar{J}_m = \bar{J} + \bar{\nabla}\times\bar{M}$$

$$\bar{\nabla}\times\underbrace{\left(\frac{\bar{B}}{\mu_0} - \bar{M}\right)}_{\bar{H}} = \bar{J}$$

$$\int_S \bar{\nabla}\times\bar{H} = \int_S \bar{J}\cdot ds$$

$$\boxed{\oint_C \bar{H}\cdot\bar{dl} = I}$$ (Ampere's law)

and

$$\bar{M} = \chi_m \bar{H}$$

→ Magnetic susceptibility

$$\bar{B} = \mu_0 (1 + \chi_m) \bar{H}$$

$$= \mu_0 \mu_r \bar{H} = \mu \bar{H} \quad \left(\frac{Wb}{m^2}\right)$$

or

$$\bar{H} = \frac{1}{\mu} \bar{B}$$

where

μ = magnetic permeability.

and

μ_r = Relative permeability.

Materials:

Diamagnetic, $\mu_r \lesssim 1$

Paramagnetics, $\mu_r \gtrsim 1$

Ferromagnetics, $\mu_r \gg 1$ (Fe)

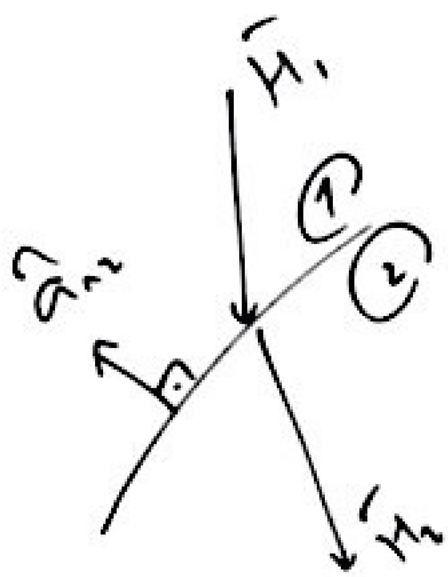
$\hookrightarrow \mu_r = 7000$

Boundary Conditions for Static Magnetic Field:

$B_{1n} = B_{2n}$ or for magnetized media

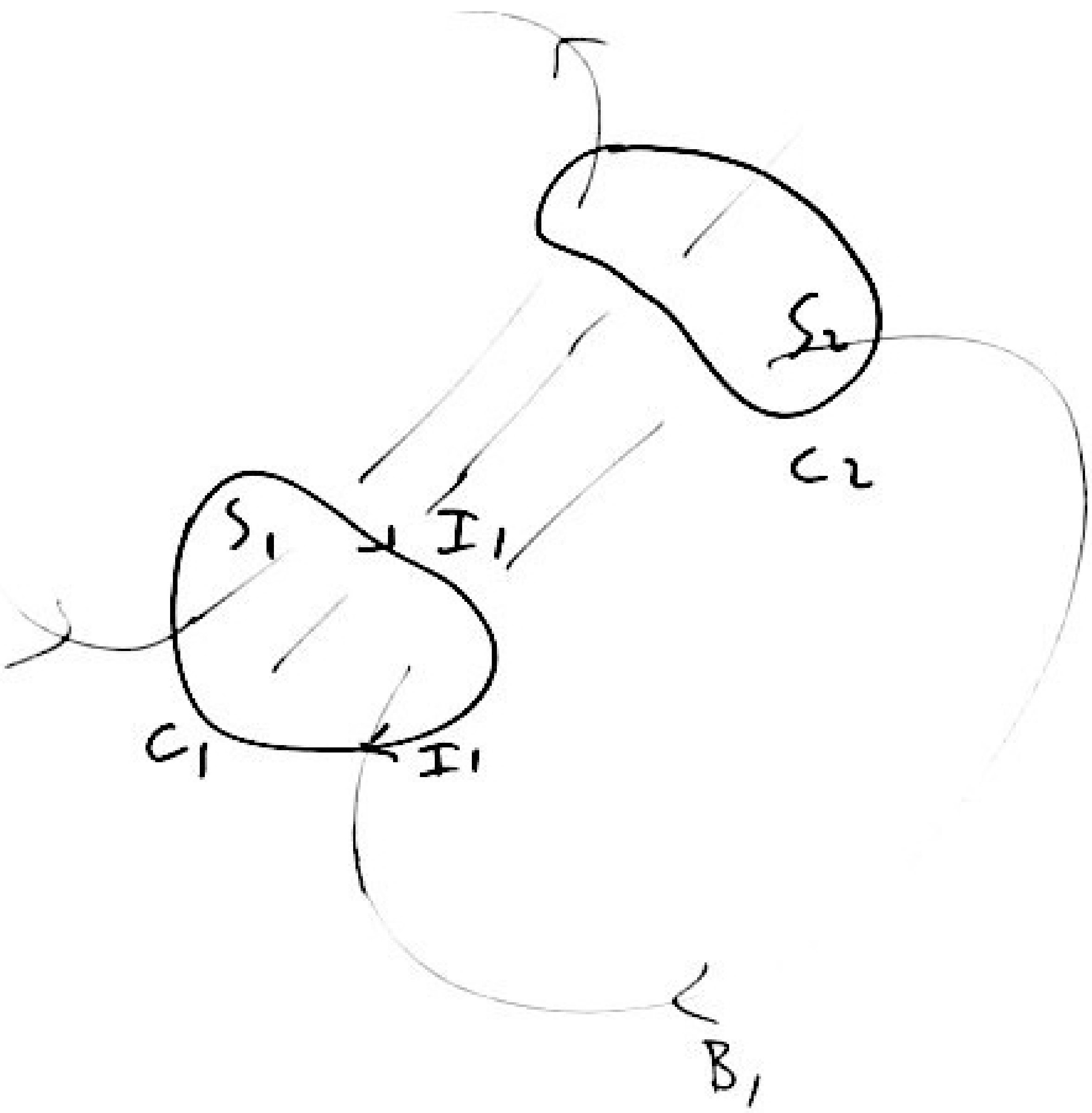
$$\boxed{\mu_1 H_{1n} = \mu_2 H_{2n}}$$ (Normal components)

and



$$\boxed{\hat{a}_{n2} \times (\bar{H}_1 - \bar{H}_2) = \bar{J}_s}$$ (Tangential components)

Inductances and Inductors:



Define

$$\phi_{12} = \int_{S_2} \bar{B}_1 \cdot d\bar{s}_2 \quad (\text{wb})$$ (flux on S_2 created by $\bar{B}_1$ on S_1 or C_1)

Since $B_1 \propto I_1$,

$$\phi_{12} \propto I_1$$

and

$$\phi_{12} = L_{12} I_1$$

when

L_{12} = Mutual inductance. (Henry)

In case C_2 has N_2 turns,

$$\Lambda_{12} = N_2 \phi_{12} \quad (wb)$$

Then

$$\Lambda_{12} = L_{12} I_1$$

or

$$\boxed{L_{12} = \frac{\Lambda_{12}}{I_1} \quad (H)}$$

(More general formula for mutual inductance.)

Self Inductance

Flux linkage per unit current in the loop C_1 itself.

$$L_{11} = \frac{\Lambda_{11}}{I_1}$$

or

In general,

$$\boxed{L = \frac{\Lambda}{I}} \ (H)$$

Steps to find Inductance (self inductance)

1-) Choose an appropriate coord. system.

2-) Assume a current I in the conducting wire.

3) Find $\vec{B}$

4-) Find $\Phi = \int_S \vec{B} \cdot d\vec{s}$

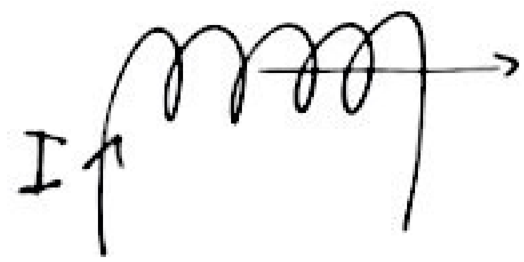
5.) Find $\Lambda = N\Phi$

6.) Find $L = \dfrac{\Lambda}{I}$.

Ex:

Find the inductance of a solenoid with iron core ($\mu_r = 7000$) having n turns.

Ans:



$\bar{B} = \hat{a}_x B_x$

$$B_x = \mu_0 \mu_r n I$$

$$\phi = \int_S \bar{B} \cdot \bar{ds}$$

S:

$$\phi = \int_0^{2\pi}\int_0^{r} \bar{B} \cdot \bar{ds}$$

$$= \underbrace{(\pi r^2)}_{S} . B_x = S B_x \ (\text{Wb})$$

$$= S \mu_0 \mu_r n I$$

$$\Lambda = n\Phi = S\mu_0\mu_r n^2 I \ (\text{wb})$$

$$L = \frac{\Lambda}{I} = S\mu_0\mu_r n^2 \ (\text{H})$$

Note that the number of turns (n) is an important parameter to adjust the inductance ($L \propto n^2$)

In order to find the magnetic field due to current carrying conductors, we have been using the Ampere's law, given by

$$\oint_C \bar{H} \cdot \bar{dl} = I$$

where I is the current passing through the surface S enclosed by the contour C.

When using the Ampere's law, the magnetic field density vector $\vec{B}$ should be constant along C so that we can take it outside the integral and solve the equation. Thus, the contour C should be selected carefully.

If a problem is given where we can <u>not</u> select such a contour, then we can <u>not</u> apply the Ampere's law. In such cases, integral techniques are used just as we did in the static electric field before.

The integral technique involves finding the vector magnetic potential first.

$$\bar{A} = \frac{\mu_0}{4\pi} \int_{v'} \frac{\bar{J}}{R}\, dv'$$

for volumetric sources.

and since $J\,dv' = J\,S\,dl' = I\,dl'$

$$A = \frac{\mu_0 I}{4\pi} \oint_{c'} \frac{dl'}{R}$$

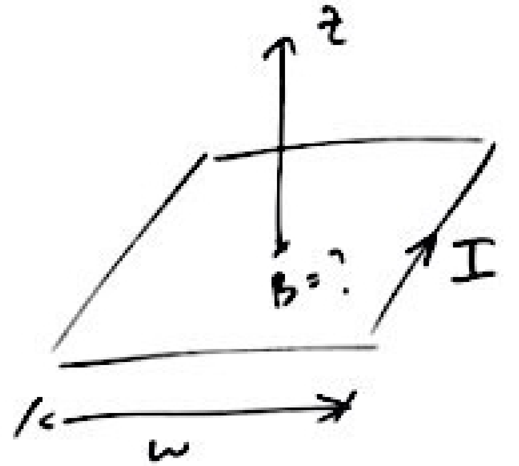
for linear sources.

Once A is obtained, the $\bar{B}$ can be evaluated from

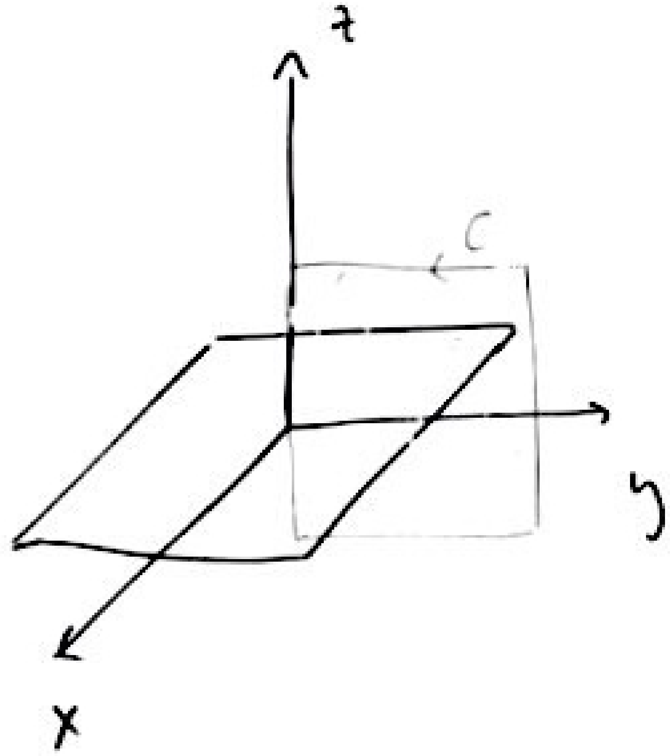
$$\bar{B} = \bar{\nabla} \times \bar{A}$$

Ex:

Find the magnetic flux density vector $\bar{B}$ at the center of a square loop with side w carrying a dc current I.



Ans:



$$\oint_C \bar{B} \cdot \bar{dl} = \mu_0 I$$

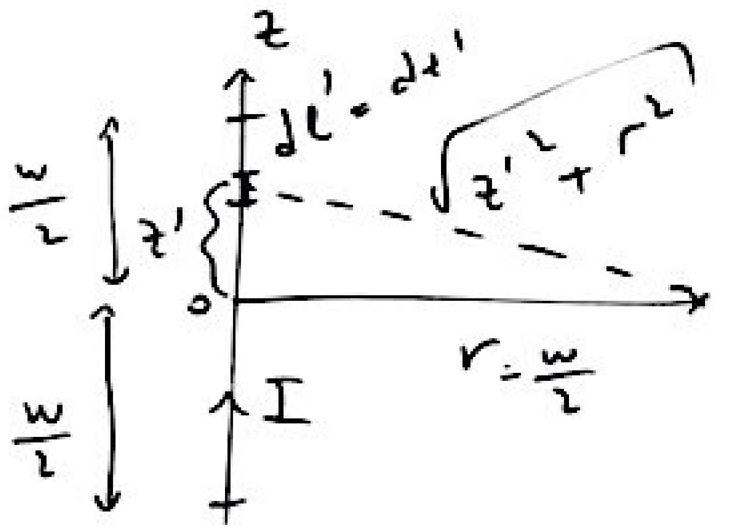
$\underbrace{\bar{B}(x,y,z)}$

$\bar{B}$ is a function of the spatial (space) coord.

Hence, we can <u>not</u> use the Ampere's law.

We will use the integral technique (the Biot Savart law)

We will find $\bar{A}$ at the center for one side only, then multiply by 4, due to symmetry, will yield the total field.



Then,

$$\bar{A} = \hat{a}_z \frac{\mu_0 I}{4\pi} \int_{-\frac{w}{2}}^{\frac{w}{2}} \frac{dz'}{\sqrt{z'^2 + r^2}}$$

or

$$\bar{A} = \hat{a}_z \frac{\mu_0 I}{4\pi} \left[\ln\left(z' + \sqrt{z'^2 + r^2}\right) \right|_{-\frac{w}{2}}^{\frac{w}{2}}$$

$$\bar{A} = \hat{a}_z \frac{\mu_0 I}{4\pi} \ln \frac{\sqrt{\left(\frac{w}{2}\right)^2 + \left(\frac{w}{2}\right)^2} + L}{\sqrt{\left(\frac{w}{2}\right)^2 + \left(\frac{w}{2}\right)^2} - L}$$

and

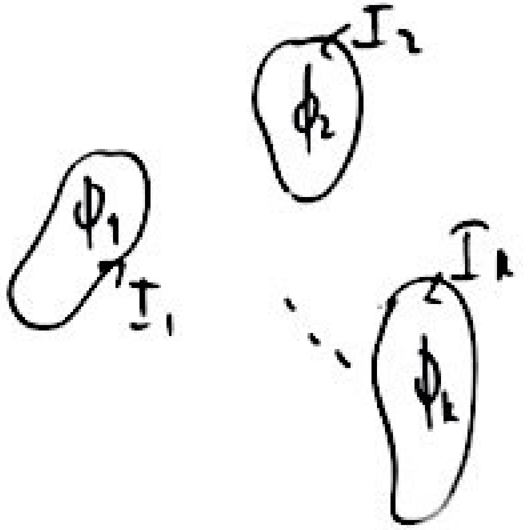
$$\bar{B} = \bar{\nabla} \times \bar{A} = \bar{\nabla} \times (\hat{a}_z A_z) = \hat{a}_r \frac{1}{r} \underbrace{\frac{\partial A_z}{\cancel{\partial \phi}}}_{\to\, 0 \text{ (due to symmetry)}} - \hat{a}_\phi \frac{\partial A_z}{\partial r}$$

$$\bar{B} = -\hat{a}_\phi \frac{\partial A_z}{\partial r} = \hat{a}_\phi \frac{\mu_0 I \frac{w}{2}}{2\pi \frac{w}{2} \sqrt{\left(\frac{w}{2}\right)^2 + \left(\frac{w}{2}\right)^2}}$$

$$\bar{B} = 4 \times \bar{B} = \hat{a}_\phi \frac{4 \mu_0 I \frac{w}{2}}{w\pi \frac{w}{2} \sqrt{2}} \left(\frac{Wb}{m^2}\right) \checkmark$$

Magnetic Energy:

$$W_m = \frac{1}{2} \sum_{k=1}^{N} I_k \phi_k$$



In terms of fields,

$$W_m = \frac{1}{2}\int_{v'} \bar{A}\cdot\bar{J}\, dv'$$

or

$$W_m = \frac{1}{2}\int_{v'} \bar{H}\cdot\bar{B}\, dv'$$

$$W_m = \frac{1}{2}\int_{v'} \frac{B^2}{\mu}\, dv'$$

$$\boxed{W_m = \frac{1}{2}\int_{v'} \mu H^2\, dv' \ (J)}$$

where

$w_m = \frac{1}{2}\mu H^2 \left(\frac{J}{m^3}\right)$ is the

energy density.

Cpt. 7 Time Varying Fields and Maxwell's Equations:

Faraday's law of Electromagnetic Induction:

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

Take the Surface integral of both sides and apply the Stoke's theorem on the LHS.

$$\boxed{\oint_c \bar{E} \cdot \bar{dl} = -\int_s \frac{\partial \bar{B}}{\partial t} \cdot ds}$$

A Stationary Circuit in a Time-Varying Mag. Field.

$$\oint_c \bar{E} \cdot d\bar{l} = -\frac{d}{dt} \int_s \bar{B} \cdot d\bar{s}$$

Since the circuit is stationary.

Then,

$$V = -\oint_c \bar{E} \cdot d\bar{l}$$

and

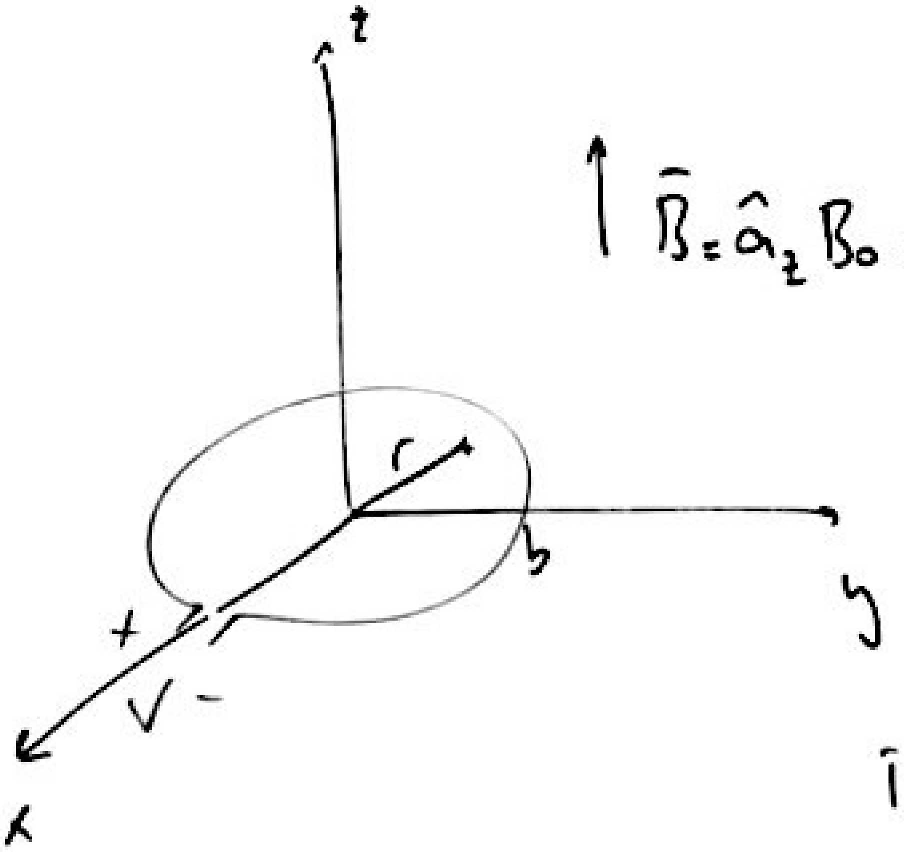
$$\phi = \int_s \bar{B} \cdot d\bar{s}$$

Then,

$$V = -\frac{d\phi}{dt}$$ (Faraday's law of electromagnetic induction)

A voltage is induced in a stationary circuit which is not moving circuit equal to the negative rate of increase in the magnetic flux linking the circuit.

Ex:



$$\bar{B} = \hat{a}_z B_0 \cos\left(\frac{\pi r}{2b}\right) \sin \omega t \left(\frac{Wb}{m^2}\right)$$

Time dependence is always sinusoidal.

Ans!

$$\phi = \int_S \bar{B} \cdot \bar{ds}$$

$$= \int_0^b \left[\hat{a}_z B_0 \cos\frac{\pi r}{2b} \sin\omega t\right] \cdot (\hat{a}_z 2\pi r\, dr)$$

$$= \frac{8b^2}{\pi}\left(\frac{\pi}{2} - 1\right) B_0 \sin\omega t$$

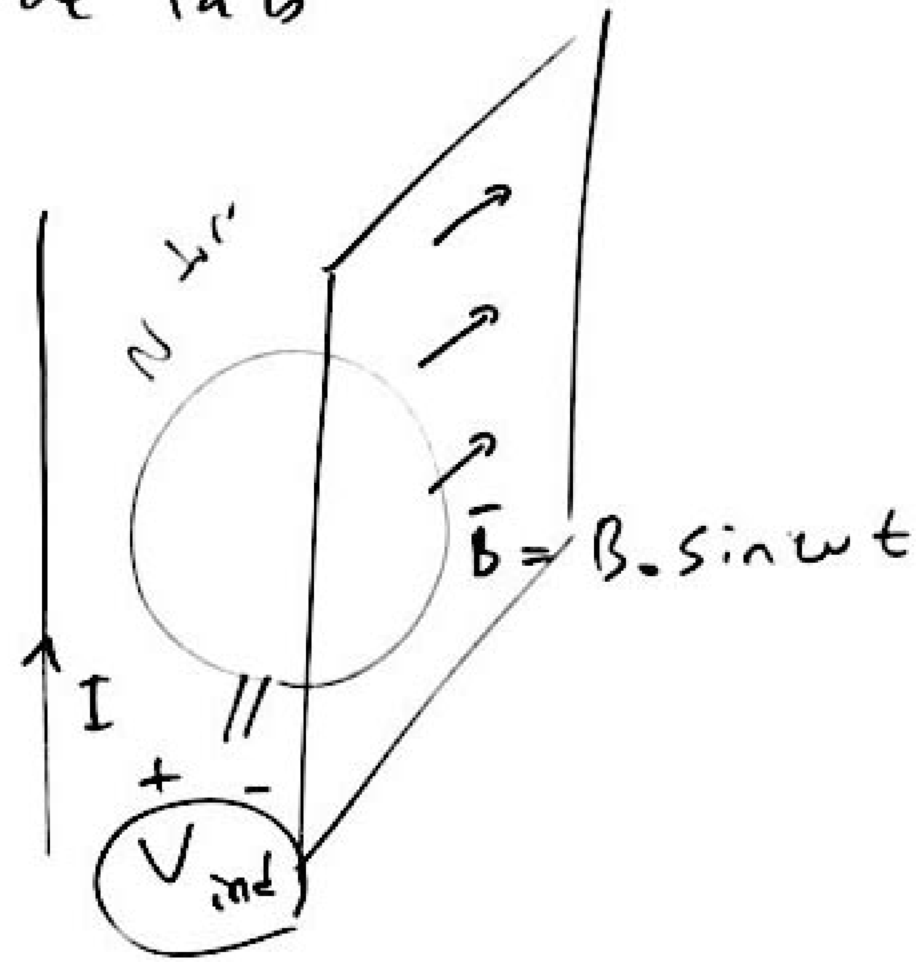
$$V = -N \cdot \frac{d\phi}{dt} \quad (\text{for } N \text{ turns})$$

$$V = -\frac{8N}{\pi} b^2 \left(\frac{\pi}{2} - 1\right) B_0 \omega \cos\omega t \;(V)$$

where w = radian frequency $= 2\pi f$

where f = Hertz freq.

In the lab



Take the Surface integral of both sides and apply the Stoke's theorem on the LHS.

$$\boxed{\oint_c \bar{E} \cdot \bar{dl} = -\int_s \frac{\partial \bar{B}}{\partial t} \cdot ds}$$